Теорија узорака

Параметри популације

$t=\sum_{i=1}^{N} x_i$	$\overline{x}=\frac{1}{N}\sum_{i=1}^{N} x_i$	$s^2=\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\overline{x})^2$	$p=\frac{A}{N}$	$s^2(p)=\frac{N}{N-1}p(1-p)$

Прост случајан узорак без понављања

$$\pi_i=\frac{\binom{N-1}{n-1}}{\binom{N}{n}}=\frac{n}{N} \qquad \pi_{ij}=\frac{\binom{N-2}{n-2}}{\binom{N}{n}}=\frac{n(n-1)}{N(N-1)}$$

θ	$\overline{x}$	t	s^2	p	A
$\hat{\theta}$	$\overline{x}_n=\frac{1}{n}\sum_{i\in S} x_i$	$\hat{t}=N\overline{x}_n$	$s_n^2=\frac{1}{n-1}\sum_{i\in S}(x_i-\overline{x}_n)^2$	$p_n=\frac{a}{n}$	$\hat{A}=Np_n$
$D(\hat{\theta})$	$\frac{s^2}{n}(1-\frac{n}{N})$	$\frac{N^2s^2}{n}(1-\frac{n}{N})$		$\frac{N-n}{n(N-1)}p(1-p)$	$\frac{N^2(N-n)}{n(N-1)}p(1-p)$
$\widehat{D(\hat{\theta})}$	$\frac{s_n^2}{n}(1-\frac{n}{N})$	$\frac{N^2s_n^2}{n}(1-\frac{n}{N})$		$\frac{N-n}{N(n-1)}p_n(1-p_n)$	$\frac{N(N-n)}{n-1}p_n(1-p_n)$

θ	I_θ ниво поверења $1-\alpha$	$n\geq$
$\overline{x}$	$\left[\overline{x}_n-z\frac{s_n}{\sqrt{n}}\sqrt{1-\frac{n}{N}},\overline{x}_n+z\frac{s_n}{\sqrt{n}}\sqrt{1-\frac{n}{N}}\right]$	$\left(\frac{d^2}{z^2s^2}+\frac{1}{N}\right)^{-1}$
$n<30$	$\left[\overline{x}_n-t\frac{s_n}{\sqrt{n}}\sqrt{1-\frac{n}{N}},\overline{x}_n+t\frac{s_n}{\sqrt{n}}\sqrt{1-\frac{n}{N}}\right]$	
t	$\left[\hat{t}-z\frac{s_n}{\sqrt{n}}\sqrt{N(N-n)},\hat{t}+z\frac{s_n}{\sqrt{n}}\sqrt{N(N-n)}\right]$	$\left(\frac{d^2}{N^2z^2s^2}+\frac{1}{N}\right)^{-1}$
$n<30$	$\left[\hat{t}-t\frac{s_n}{\sqrt{n}}\sqrt{N(N-n)},\hat{t}+t\frac{s_n}{\sqrt{n}}\sqrt{N(N-n)}\right]$	
p	$\left[p_n-z\sqrt{\frac{N-n}{N(n-1)}p_n(1-p_n)}-\frac{1}{2n},p_n+z\sqrt{\frac{N-n}{N(n-1)}p_n(1-p_n)}+\frac{1}{2n}\right]$	$\left(\frac{(N-1)d^2}{Nz^2p(1-p)}+\frac{1}{N}\right)^{-1}$
$n<30$	$\left[p_n-t\sqrt{\frac{N-n}{N(n-1)}p_n(1-p_n)}-\frac{1}{2n},p_n+t\sqrt{\frac{N-n}{N(n-1)}p_n(1-p_n)}+\frac{1}{2n}\right]$	
p	$0\leq p(1-p)\leq\frac{1}{4}\Longrightarrow$	$\left(\frac{4(N-1)d^2}{Nz^2}+\frac{1}{N}\right)^{-1}$

$$F_Z(z)=1-\frac{\alpha}{2} \qquad F_{t_{n-1}}(t)=1-\frac{\alpha}{2}$$

Прост случајан узорак са понављањем

θ	$\overline{x}$	t	s^2	p	A
$\hat{\theta}$	$\overline{x}_n=\frac{1}{n}\sum_{i\in S} x_i$	$\hat{t}=N\overline{x}_n$	$\frac{N}{N-1}s_n^2$	$p_n=\frac{a}{n}$	$\hat{A}=Np_n$
$D(\hat{\theta})$	$\frac{N-1}{Nn}s^2$	$\frac{N(N-1)}{n}s^2$		$\frac{p(1-p)}{n}$	$\frac{N^2p(1-p)}{n}$
$\widehat{D(\hat{\theta})}$	$\frac{s_n^2}{n}$	$\frac{N^2s_n^2}{n}$		$\frac{p_n(1-p_n)}{n-1}$	$\frac{N^2p_n(1-p_n)}{n-1}$

Узорковање са неједнаким вероватноћама избора

Hansen-Hurwitz-ове оцене

θ	t	$\overline{x}$
$\hat{\theta}$	$\hat{t}_{HH} = \frac{1}{n}\sum_{i\in S}\frac{x_i}{p_i}$	$\overline{x}_{HH} = \frac{1}{Nn}\sum_{i\in S}\frac{x_i}{p_i}$
$D(\hat{\theta})$	$\frac{1}{n}\sum_{i=1}^{N} p_i\big(\frac{x_i}{p_i} - t\big)^2$	$\frac{1}{n}\sum_{i=1}^{N} p_i\big(\frac{x_i}{Np_i} - \overline{x}\big)^2$
$\widehat{D(\hat{\theta})}$	$\frac{1}{n(n-1)}\sum_{i\in S}\big(\frac{x_i}{p_i} - \hat{t}_{HH}\big)^2$	$\frac{1}{n(n-1)}\sum_{i\in S}\big(\frac{x_i}{Np_i} - \overline{x}_{HH}\big)^2$

Horvitz-Thompson-ове оцене

θ	t	$\overline{x}$
$\hat{\theta}$	$\hat{t}_{HT} = \sum_{i=1}^{\nu}\frac{x_i}{\pi_i}$	$\overline{x}_{HT} = \frac{1}{N}\sum_{i=1}^{\nu}\frac{x_i}{\pi_i}$
$D(\hat{\theta})$	$\sum_{i=1}^{N}\frac{1-\pi_i}{\pi_i}{x_i}^2 + \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j}x_ix_j$	$\frac{1}{N^2}\Big(\sum_{i=1}^{N}\frac{1-\pi_i}{\pi_i}{x_i}^2 + \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j}x_ix_j\Big)$
$\widehat{D(\hat{\theta})}$	$\sum_{i=1}^{\nu}\frac{1-\pi_i}{\pi_i^2}{x_i}^2 + \sum_{i=1}^{\nu}\sum_{\substack{j=1\\ j\neq i}}^{\nu}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j}\frac{x_ix_j}{\pi_{ij}}$	$\frac{1}{N^2}\Big(\sum_{i=1}^{\nu}\frac{1-\pi_i}{\pi_i^2}{x_i}^2 + \sum_{i=1}^{\nu}\sum_{\substack{j=1\\ j\neq i}}^{\nu}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j}\frac{x_ix_j}{\pi_{ij}}\Big)$
$\widehat{D_{syg}(\hat{\theta})}$	$\sum_{i\in S}\sum_{\substack{j\in S\\ j>i}}\frac{\pi_i\pi_j-\pi_{ij}}{\pi_{ij}}\Big(\frac{x_i}{\pi_i} - \frac{x_j}{\pi_j}\Big)^2$	$\frac{1}{N^2}\sum_{i\in S}\sum_{\substack{j\in S\\ j>i}}\frac{\pi_i\pi_j-\pi_{ij}}{\pi_{ij}}\Big(\frac{x_i}{\pi_i} - \frac{x_j}{\pi_j}\Big)^2$

Стратификован случајан узорак без понављања

$\overline{x}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$	$s_h^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \overline{x}_h)^2$	$p_h = \frac{A_h}{N_h}$
$\overline{x}_{n_h} = \frac{1}{n_h}\sum_{i\in S_h} x_{hi}$	$s_{n_h}^2 = \frac{1}{n_h-1}\sum_{i\in S_h}(x_{hi} - \overline{x}_{n_h})^2$	$p_{n_h} = \frac{a_h}{n_h}$

θ	t	$\overline{x}$	p
$\hat{\theta}$	$\hat{t}_{str} = \sum_{h=1}^{L} N_h\overline{x}_{n_h}$	$\overline{x}_{str} = \frac{1}{N}\sum_{h=1}^{L} N_h\overline{x}_{n_h}$	$\hat{p}_{str} = \frac{1}{N}\sum_{h=1}^{L} N_hp_{n_h}$
$D(\hat{\theta})$	$\sum_{h=1}^{L}\frac{N_h^2s_h^2}{n_h}\big(1 - \frac{n_h}{N_h}\big)$	$\frac{1}{N^2}\sum_{h=1}^{L}\frac{N_h^2s_h^2}{n_h}\big(1 - \frac{n_h}{N_h}\big)$	$\frac{1}{N^2}\sum_{h=1}^{L}\frac{N_h^2(N_h-n_h)}{n_h(N_h-1)}p_h(1 - p_h)$
$\widehat{D(\hat{\theta})}$	$\sum_{h=1}^{L}\frac{N_h^2s_{n_h}^2}{n_h}\big(1 - \frac{n_h}{N_h}\big)$	$\frac{1}{N^2}\sum_{h=1}^{L}\frac{N_h^2s_{n_h}^2}{n_h}\big(1 - \frac{n_h}{N_h}\big)$	$\frac{1}{N^2}\sum_{h=1}^{L}\frac{N_h(N_h-n_h)}{n_h-1}p_{n_h}(1 - p_{n_h})$

Методи за одређивање обима узорка по стратумима

пропорционални	оптимални	оптимални $Neyman$-ов
	$C = c_0 + \sum_{h=1}^{L} c_h n_h$	$C = c_0 + cn$
$n_h = \frac{n}{N} N_h$	$n_h = \frac{(C-c_0)\frac{N_h s_h}{\sqrt{c_h}}}{\sum_{k=1}^{L} \sqrt{c_k} N_k s_k}$	$n_h = \frac{N_h s_h}{\sum_{k=1}^{L} N_k s_k} n$
$D(\hat{t}_{prop}) = \frac{N}{n}(1-\frac{n}{N}) \sum_{h=1}^{L} N_h s_h^2$		$D(\hat{t}_{opt}) = \frac{1}{n}\Big(\sum_{h=1}^{L} N_h s_h\Big)^2 - \sum_{h=1}^{L} N_h s_h^2$

Стратификован случајан узорак са понављањем

θ	t	$\overline{x}$	p
$\hat{\theta}$	$\hat{t}_{str} = \sum_{h=1}^{L} N_h \overline{x}_{n_h}$	$\overline{x}_{str} = \frac{1}{N} \sum_{h=1}^{L} N_h \overline{x}_{n_h}$	$\hat{p}_{str} = \frac{1}{N} \sum_{h=1}^{L} N_h p_{n_h}$
$D(\hat{\theta})$	$\sum_{h=1}^{L} \frac{N_h(N_h-1)s_h^2}{n_h}$	$\frac{1}{N^2} \sum_{h=1}^{L} \frac{N_h(N_h-1)s_h^2}{n_h}$	$\frac{1}{N^2} \sum_{h=1}^{L} \frac{N_h^2 p_h(1-p_h)}{n_h}$
$\widehat{D(\hat{\theta})}$	$\sum_{h=1}^{L} \frac{N_h^2 s_{n_h}^2}{n_h}$	$\frac{1}{N^2} \sum_{h=1}^{L} \frac{N_h^2 s_{n_h}^2}{n_h}$	$\frac{1}{N^2} \sum_{h=1}^{L} \frac{N_h^2 p_{n_h}(1-p_{n_h})}{n_h-1}$

Количничко оцењивање на основу простог случајног узорка без понављања

θ	R	t	$\overline{x}$
$\hat{\theta}$	$\hat{R} = \frac{\overline{x}_n}{\overline{y}_n}$	$\hat{t}_R = \hat{R} t_y$	$\overline{x}_R = \hat{R}\overline{y}$
$D(\hat{\theta}) \approx$	$\frac{1-\frac{n}{N}}{n\overline{y}^2} \frac{1}{N-1} \sum_{i=1}^{N} (x_i - R y_i)^2$	$\frac{N^2(1-\frac{n}{N})}{n} \frac{1}{N-1} \sum_{i=1}^{N} (x_i - R y_i)^2$	$\frac{1-\frac{n}{N}}{n} \frac{1}{N-1} \sum_{i=1}^{N} (x_i - R y_i)^2$
$\widehat{D(\hat{\theta})} \approx$	$\frac{(1-\frac{n}{N})}{n\overline{y}^2} \frac{1}{n-1} \sum_{i\in S} (x_i - \hat{R} y_i)^2$	$\frac{N^2(1-\frac{n}{N})}{n} \frac{1}{n-1} \sum_{i\in S} (x_i - \hat{R} y_i)^2$	$\frac{(1-\frac{n}{N})}{n} \frac{1}{n-1} \sum_{i\in S} (x_i - \hat{R} y_i)^2$

$$R = \frac{t}{t_y} = \frac{\overline{x}}{\overline{y}} \qquad \rho = \frac{\sum_{i=1}^{N} (x_i-\overline{x})(y_i-\overline{y})}{(N-1)s_x s_y} \qquad \frac{1}{N-1} \sum_{i=1}^{N} (x_i - R y_i)^2 = s_x^2 + R^2 s_y^2 - 2R\rho s_x s_y$$

Количничко оцењивање на основу узорковања јединки са неједнаким вероватноћама избора

θ	R	t	$\overline{x}$
$\hat{\theta}$	$\hat{R} = \frac{\hat{t}_{HT}(x)}{\hat{t}_{HT}(y)} = \frac{\sum_{i=1}^{\nu} \frac{x_i}{\pi_i}}{\sum_{i=1}^{\nu} \frac{y_i}{\pi_i}}$	$\hat{t}_R = \hat{R} t_y$	$\overline{x}_R = \hat{R}\overline{y}$
$D(\hat{\theta}) \approx$	$\frac{1}{t_y^2}\Big[\sum_{i=1}^{N} \frac{1-\pi_i}{\pi_i}(x_i - R y_i)^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j}(x_i - R y_i)(x_j - R y_j)\Big]$	$t_y^2 D(\hat{R})$	$\overline{y}^2 D(\hat{R})$
$\widehat{D(\hat{\theta})} \approx$	$\frac{1}{t_y^2}\Big[\sum_{i=1}^{\nu} \frac{1-\pi_i}{\pi_i^2}(x_i - \hat{R} y_i)^2 + \sum_{i=1}^{\nu} \sum_{\substack{j=1 \\ j\neq i}}^{\nu} \frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j} \frac{(x_i-\hat{R}y_i)(x_j-\hat{R}y_j)}{\pi_{ij}}\Big]$	$t_y^2 \widehat{D(\hat{R})}$	$\overline{y}^2 \widehat{D(\hat{R})}$

Количничко оцењивање на основу стратификованог случајног узорка

Комбиноване оцене

θ	R	t	$\overline{x}$
$\hat{\theta}$	$\hat{R}_c = \frac{\overline{x}_{str}}{\overline{y}_{str}}$	$\hat{t}_{Rc} = \hat{R}_c t_y$	$\overline{x}_{Rc} = \hat{R}_c \overline{y}$
$D(\hat{\theta}) \approx$	$\frac{1}{t_y^2}\Big[\sum\limits_{h=1}^{L} \frac{N_h^2}{n_h}(1-\frac{n_h}{N_h})\big(s_h^2(x) + R^2 s_h^2(y) - 2R\rho_h s_h(x)s_h(y)\big)\Big]$	$t_y^2 D(\hat{R}_c)$	$\overline{y}^2 D(\hat{R}_c)$

Посебне оцене

θ	t	$\overline{x}$
$\hat{\theta}$	$\hat{t}_{Rs} = \sum\limits_{h=1}^{L} \hat{R}_h t_h(y) = \sum\limits_{h=1}^{L} \frac{\overline{x}_{n_h}}{\overline{y}_{n_h}} t_h(y)$	$\overline{x}_{Rs} = \frac{1}{N}\hat{t}_{Rs}$
$D(\hat{\theta}) \approx$	$\sum\limits_{h=1}^{L} \frac{N_h^2}{n_h}(1-\frac{n_h}{N_h})\big(s_h^2(x) + R_h^2 s_h^2(y) - 2R_h\rho_h s_h(x)s_h(y)\big)$	$\frac{1}{N^2}D(\hat{t}_{Rs})$

Регресионо оцењивање на основу простог случајног узорка без понављања

	$b = b_0$	$\hat{b} = \hat{\rho}\frac{s_n(x)}{s_n(y)}$	
θ	$\overline{x}$	$\overline{x}$	t
$\hat{\theta}$	$\overline{x}_{LR} = \overline{x}_n + b_0(\overline{y} - \overline{y_n})$	$\overline{x}_{LR} = \overline{x}_n + \hat{b}(\overline{y} - \overline{y_n})$	$\hat{t}_{LR} = N\overline{x}_{LR}$
$D(\hat{\theta})$	$\big(1-\frac{n}{N}\big)\frac{1}{n}(s_x^2 + b_0^2 s_y^2 - 2b_0\rho s_x s_y)$	$\approx \big(1-\frac{n}{N}\big)\frac{1}{n}s_x^2(1-\rho^2)$	$N^2 D(\overline{x}_{LR})$
$\widehat{D(\hat{\theta})}$	$\big(1-\frac{n}{N}\big)\frac{1}{n}\big(s_n^2(x) + b_0^2 s_n^2(y) - 2b_0\hat{\rho}s_n(x)s_n(y)\big)$		$N^2\widehat{D(\overline{x}_{LR})}$

Регресионо оцењивање на основу стратификованог случајног узорка

Комбиноване оцене

	$b = b_0$	$\hat{b} = \frac{\sum\limits_{h=1}^{L}\left(\frac{N_h^2\left(1-\frac{n_h}{N_h}\right)}{n_h(n_h-1)}\sum\limits_{i\in S_h}(x_{hi}-\overline{x}_{n_h})(y_{hi}-\overline{y}_{n_h})\right)}{\sum\limits_{h=1}^{L}\left(\frac{N_h^2\left(1-\frac{n_h}{N_h}\right)}{n_h(n_h-1)}\sum\limits_{i\in S_h}(y_{hi}-\overline{y}_{n_h})^2\right)}$
θ	$\overline{x}$	$\overline{x}$
$\hat{\theta}$	$\overline{x}_{LRc} = \overline{x}_{str} + b_0(\overline{y} - \overline{y}_{str})$	$\overline{x}_{LRc} = \overline{x}_{str} + \hat{b}(\overline{y} - \overline{y}_{str})$
$D(\hat{\theta})$	$\frac{1}{N^2}\sum\limits_{h=1}^{L}\frac{N_h^2\left(1-\frac{n_h}{N_h}\right)}{n_h}\big(s_h^2(x)+b_0^2 s_h^2(y)-2b_0\rho_h s_h(x)s_h(y)\big)$	
$\widehat{D(\hat{\theta})}$	$\frac{1}{N^2}\sum\limits_{h=1}^{L}\frac{N_h^2\left(1-\frac{n_h}{N_h}\right)}{n_h}\big(s_{n_h}^2(x)+b_0^2 s_{n_h}^2(y)-2b_0\hat{\rho}_h s_{n_h}(x)s_{n_h}(y)\big)$	

Посебне оцене

	$b_h = b_{h0}$	$\hat{b}_h = \hat{\rho}_h \frac{s_{n_h}(x)}{s_{n_h}(y)}$
θ	$\overline{x}$	$\overline{x}$
$\hat{\theta}$	$\overline{x}_{LRs} = \frac{1}{N}\sum_{h=1}^{L} N_h(\overline{x}_{n_h} + b_{h0}(\overline{y}_h - \overline{y}_{n_h}))$	$\overline{x}_{LRs} = \frac{1}{N}\sum_{h=1}^{L} N_h(\overline{x}_{n_h} + \hat{b}_h(\overline{y}_h - \overline{y}_{n_h}))$
$D(\hat{\theta})$	$\frac{1}{N^2}\sum_{h=1}^{L}\frac{N_h^2\left(1-\frac{n_h}{N_h}\right)}{n_h}\left(s_h^2(x)+b_{h0}^2 s_h^2(y)-2b_{h0}\rho_h s_h(x)s_h(y)\right)$	$\approx \frac{1}{N^2}\sum_{h=1}^{L}\frac{N_h^2\left(1-\frac{n_h}{N_h}\right)}{n_h} s_h^2(x)(1-\rho_h^2)$
$\widehat{D(\hat{\theta})}$	$\frac{1}{N^2}\sum_{h=1}^{L}\frac{N_h^2\left(1-\frac{n_h}{N_h}\right)}{n_h}\left(s_{n_h}^2(x)+b_{h0}^2 s_{n_h}^2(y)-2b_{h0}\hat{\rho}_h s_{n_h}(x)s_{n_h}(y)\right)$	

Кластер узорак код кога се примарне јединице бирају као прост случајан узорак

$t_i = \sum_{j=1}^{M_i} x_{ij}$	$t = \sum_{i=1}^{N} t_i$	$R = \frac{\sum_{i=1}^{N} t_i}{\sum_{i=1}^{N} M_i}$	$\hat{R} = \frac{\sum_{i\in S} t_i}{\sum_{i\in S} M_i}$

θ	t	$\overline{x}$	t	$\overline{x}$
$\hat{\theta}$	$\hat{t} = \frac{N}{n}\sum_{i\in S} t_i$	$\frac{\hat{t}}{\sum_{i=1}^{N} M_i}$	$\hat{t}_R = \frac{\sum_{i\in S} t_i}{\sum_{i\in S} M_i}\sum_{i=1}^{N} M_i$	$\overline{x}_R = \frac{\sum_{i\in S} t_i}{\sum_{i\in S} M_i}$
$D(\hat{\theta})$	$N^2(1-\frac{n}{N})\frac{1}{n}\frac{1}{N-1}\sum_{i=1}^{N}(t_i-\frac{t}{N})^2$	$\frac{D(\hat{t})}{\left(\sum_{i=1}^{N} M_i\right)^2}$	$\approx N^2(1-\frac{n}{N})\frac{1}{n}\frac{1}{N-1}\sum_{i=1}^{N}(t_i-RM_i)^2$	$\approx \frac{D(\hat{t}_R)}{\left(\sum_{i=1}^{N} M_i\right)^2}$
$\widehat{D(\hat{\theta})}$	$N^2(1-\frac{n}{N})\frac{1}{n}\frac{1}{n-1}\sum_{i\in S}\left(t_i-\frac{1}{n}\sum_{j\in S} t_j\right)^2$	$\frac{\widehat{D(\hat{t})}}{\left(\sum_{i=1}^{N} M_i\right)^2}$	$\approx N^2(1-\frac{n}{N})\frac{1}{n}\frac{1}{n-1}\sum_{i\in S}(t_i-\hat{R}M_i)^2$	$\approx \frac{\widehat{D(\hat{t}_R)}}{\left(\sum_{i=1}^{N} M_i\right)^2}$

Кластери са по M секундарних јединица

$$\rho_{uk} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{\substack{k=1\\k\neq j}}^{M}(x_{ij}-\overline{x})(x_{ik}-\overline{x})}{(M-1)(NM-1)s^2} \qquad D(\hat{t}) = N^2\left(1-\frac{n}{N}\right)\frac{1}{n}\frac{NM-1}{N-1}s^2(1+(M-1)\rho_{uk})$$

Кластер узорак код кога се примарне јединице бирају са вероватноћама пропорционалним величини

θ	t	$\overline{x}$
$\hat{\theta}$	$\hat{t}_{HH} = \frac{\sum_{i=1}^{N} M_i}{n}\sum_{i\in S}\frac{t_i}{M_i}$	$\overline{x}_{HH} = \frac{1}{n}\sum_{i\in S}\frac{t_i}{M_i}$
$D(\hat{\theta})$	$\frac{\sum_{i=1}^{N} M_i}{n}\sum_{i=1}^{N} M_i\left(\frac{t_i}{M_i}-\overline{x}\right)^2$	$\frac{1}{\sum_{i=1}^{N} M_i}\frac{1}{n}\sum_{i=1}^{N} M_i\left(\frac{t_i}{M_i}-\overline{x}\right)^2$
$\widehat{D(\hat{\theta})}$	$\frac{1}{n(n-1)}\sum_{i\in S}\left(\frac{t_i}{M_i}\sum_{j=1}^{N} M_j - \hat{t}_{HH}\right)^2$	$\frac{1}{n(n-1)}\sum_{i\in S}\left(\frac{t_i}{M_i} - \overline{x}_{HH}\right)^2$

θ	t	$\overline{x}$
$\hat{\theta}$	$\hat{t}_{HT} = \sum_{i=1}^{\nu} \frac{t_i}{\pi_i}$	$\overline{x}_{HT} = \frac{1}{\sum_{i=1}^{N} M_i} \hat{t}_{HT}$
$D(\hat{\theta})$	$\sum_{i=1}^{N} \frac{1-\pi_i}{\pi_i} t_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} t_i t_j$	$\frac{1}{\left(\sum_{i=1}^{N} M_i\right)^2} D(\hat{t}_{HT})$
$\widehat{D(\hat{\theta})}$	$\sum_{i=1}^{\nu} \frac{1-\pi_i}{\pi_i^2} t_i^2 + \sum_{i=1}^{\nu} \sum_{\substack{j=1 \\ j \neq i}}^{\nu} \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} \frac{t_i t_j}{\pi_{ij}}$	$\frac{1}{\left(\sum_{i=1}^{N} M_i\right)^2} \widehat{D(\hat{t}_{HT})}$

Систематски узорак

$$N = nk \qquad s_{sis}^2 = \frac{1}{k(n-1)} \sum_{i=1}^{k} \sum_{j=1}^{n} (x_{ij} - \overline{x}_i)^2$$

θ	$\overline{x}$	t
$\hat{\theta}$	$\overline{x}_{sis} = \frac{1}{n} \sum_{i \in S} x_i$	$\hat{t}_{sis} = N \overline{x}_{sis}$
$D(\hat{\theta})$	$\frac{1}{k} \sum_{i=1}^{k} (\overline{x}_i - \overline{x})^2 = \frac{N-1}{N} s^2 - \frac{k(n-1)}{N} s_{sis}^2 = \frac{N-1}{N} \frac{s^2}{n} \big(1 + (n-1)\rho_{uk}\big)$	$N^2 D(\overline{x}_{sis})$

Линеарни тренд $x_i = a + bi$

$$D(\overline{x}_n) = \frac{(k-1)(N+1)}{12} b^2 \qquad D(\overline{x}_{sis}) = \frac{k^2-1}{12} b^2 \qquad D(\overline{x}_{str}) = \frac{k^2-1}{12n} b^2$$