

Теорија узорака

Параметри популације

$t = \sum_{i=1}^N x_i$	$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$	$s^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2$	$p = \frac{A}{N}$	$s^2(p) = \frac{N}{N-1} p(1-p)$
------------------------	--	--	-------------------	---------------------------------

Прост случајан узорак без понављања

$$\pi_i = \frac{\binom{N-1}{n-1}}{\binom{N}{n}} = \frac{n}{N} \qquad \pi_{ij} = \frac{\binom{N-2}{n-2}}{\binom{N}{n}} = \frac{n(n-1)}{N(N-1)}$$

θ	$\bar{x}$	t	s^2	p	A
$\hat{\theta}$	$\bar{x}_n = \frac{1}{n} \sum_{i \in S} x_i$	$\hat{t} = N\bar{x}_n$	$s_n^2 = \frac{1}{n-1} \sum_{i \in S} (x_i - \bar{x}_n)^2$	$p_n = \frac{a}{n}$	$\hat{A} = Np_n$
$D(\hat{\theta})$	$\frac{s_n^2}{n} (1 - \frac{n}{N})$	$\frac{N^2 s_n^2}{n} (1 - \frac{n}{N})$		$\frac{N-n}{n(N-1)} p(1-p)$	$\frac{N^2(N-n)}{n(N-1)} p(1-p)$
$\widehat{D(\hat{\theta})}$	$\frac{s_n^2}{n} (1 - \frac{n}{N})$	$\frac{N^2 s_n^2}{n} (1 - \frac{n}{N})$		$\frac{N-n}{N(n-1)} p_n(1-p_n)$	$\frac{N(N-n)}{n-1} p_n(1-p_n)$

θ	I_θ ниво поверења $1 - \alpha$	$n \geq$
$\bar{x}$ $n < 30$	$\left[\bar{x}_n - z \frac{s_n}{\sqrt{n}} \sqrt{1 - \frac{n}{N}}, \bar{x}_n + z \frac{s_n}{\sqrt{n}} \sqrt{1 - \frac{n}{N}} \right]$ $\left[\bar{x}_n - t \frac{s_n}{\sqrt{n}} \sqrt{1 - \frac{n}{N}}, \bar{x}_n + t \frac{s_n}{\sqrt{n}} \sqrt{1 - \frac{n}{N}} \right]$	$\left(\frac{d^2}{z^2 s^2} + \frac{1}{N} \right)^{-1}$
t $n < 30$	$\left[\hat{t} - z \frac{s_n}{\sqrt{n}} \sqrt{N(N-n)}, \hat{t} + z \frac{s_n}{\sqrt{n}} \sqrt{N(N-n)} \right]$ $\left[\hat{t} - t \frac{s_n}{\sqrt{n}} \sqrt{N(N-n)}, \hat{t} + t \frac{s_n}{\sqrt{n}} \sqrt{N(N-n)} \right]$	$\left(\frac{d^2}{N^2 z^2 s^2} + \frac{1}{N} \right)^{-1}$
p $n < 30$	$\left[p_n - z \sqrt{\frac{N-n}{N(n-1)} p_n(1-p_n)} - \frac{1}{2n}, p_n + z \sqrt{\frac{N-n}{N(n-1)} p_n(1-p_n)} + \frac{1}{2n} \right]$ $\left[p_n - t \sqrt{\frac{N-n}{N(n-1)} p_n(1-p_n)} - \frac{1}{2n}, p_n + t \sqrt{\frac{N-n}{N(n-1)} p_n(1-p_n)} + \frac{1}{2n} \right]$	$\left(\frac{(N-1)d^2}{N z^2 p(1-p)} + \frac{1}{N} \right)^{-1}$
p	$0 \leq p(1-p) \leq \frac{1}{4} \implies$	$\left(\frac{4(N-1)d^2}{N z^2} + \frac{1}{N} \right)^{-1}$

$$F_Z(z) = 1 - \frac{\alpha}{2} \qquad F_{t_{n-1}}(t) = 1 - \frac{\alpha}{2}$$

Прост случајан узорак са понављањем

θ	$\bar{x}$	t	s^2	p	A
$\hat{\theta}$	$\bar{x}_n = \frac{1}{n} \sum_{i \in S} x_i$	$\hat{t} = N\bar{x}_n$	$\frac{N}{N-1} s_n^2$	$p_n = \frac{a}{n}$	$\hat{A} = Np_n$
$D(\hat{\theta})$	$\frac{N-1}{Nn} s^2$	$\frac{N(N-1)}{n} s^2$		$\frac{p(1-p)}{n}$	$\frac{N^2 p(1-p)}{n}$
$\widehat{D(\hat{\theta})}$	$\frac{s_n^2}{n}$	$\frac{N^2 s_n^2}{n}$		$\frac{p_n(1-p_n)}{n-1}$	$\frac{N^2 p_n(1-p_n)}{n-1}$

Узорковање са неједнаким вероватноћама избора

Hansen-Hurwitz-ове оцене

θ	t	$\bar{x}$
$\hat{\theta}$	$\hat{t}_{HH} = \frac{1}{n} \sum_{i \in S} \frac{x_i}{p_i}$	$\bar{x}_{HH} = \frac{1}{Nn} \sum_{i \in S} \frac{x_i}{p_i}$
$D(\hat{\theta})$	$\frac{1}{n} \sum_{i=1}^N p_i \left(\frac{x_i}{p_i} - t \right)^2$	$\frac{1}{n} \sum_{i=1}^N p_i \left(\frac{x_i}{Np_i} - \bar{x} \right)^2$
$\widehat{D(\hat{\theta})}$	$\frac{1}{n(n-1)} \sum_{i \in S} \left(\frac{x_i}{p_i} - \hat{t}_{HH} \right)^2$	$\frac{1}{n(n-1)} \sum_{i \in S} \left(\frac{x_i}{Np_i} - \bar{x}_{HH} \right)^2$

Horvitz-Thompson-ове оцене

θ	t	$\bar{x}$
$\hat{\theta}$	$\hat{t}_{HT} = \sum_{i=1}^{\nu} \frac{x_i}{\pi_i}$	$\bar{x}_{HT} = \frac{1}{N} \sum_{i=1}^{\nu} \frac{x_i}{\pi_i}$
$D(\hat{\theta})$	$\sum_{i=1}^N \frac{1-\pi_i}{\pi_i} x_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} x_i x_j$	$\frac{1}{N^2} \left(\sum_{i=1}^N \frac{1-\pi_i}{\pi_i} x_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} x_i x_j \right)$
$\widehat{D(\hat{\theta})}$	$\sum_{i=1}^{\nu} \frac{1-\pi_i}{\pi_i^2} x_i^2 + \sum_{i=1}^{\nu} \sum_{\substack{j=1 \\ j \neq i}}^{\nu} \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} \frac{x_i x_j}{\pi_{ij}}$	$\frac{1}{N^2} \left(\sum_{i=1}^{\nu} \frac{1-\pi_i}{\pi_i^2} x_i^2 + \sum_{i=1}^{\nu} \sum_{\substack{j=1 \\ j \neq i}}^{\nu} \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} \frac{x_i x_j}{\pi_{ij}} \right)$
$\widehat{D_{syg}(\hat{\theta})}$	$\sum_{i \in S} \sum_{\substack{j \in S \\ j > i}} \frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}} \left(\frac{x_i}{\pi_i} - \frac{x_j}{\pi_j} \right)^2$	$\frac{1}{N^2} \sum_{i \in S} \sum_{\substack{j \in S \\ j > i}} \frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}} \left(\frac{x_i}{\pi_i} - \frac{x_j}{\pi_j} \right)^2$

Стратификован случајан узорак без понављања

$\bar{x}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi}$	$s_h^2 = \frac{1}{N_h-1} \sum_{i=1}^{N_h} (x_{hi} - \bar{x}_h)^2$	$p_h = \frac{A_h}{N_h}$
$\bar{x}_{n_h} = \frac{1}{n_h} \sum_{i \in S_h} x_{hi}$	$s_{n_h}^2 = \frac{1}{n_h-1} \sum_{i \in S_h} (x_{hi} - \bar{x}_{n_h})^2$	$p_{n_h} = \frac{a_h}{n_h}$

θ	t	$\bar{x}$	p
$\hat{\theta}$	$\hat{t}_{str} = \sum_{h=1}^L N_h \bar{x}_{n_h}$	$\bar{x}_{str} = \frac{1}{N} \sum_{h=1}^L N_h \bar{x}_{n_h}$	$\hat{p}_{str} = \frac{1}{N} \sum_{h=1}^L N_h p_{n_h}$
$D(\hat{\theta})$	$\sum_{h=1}^L \frac{N_h^2 s_h^2}{n_h} \left(1 - \frac{n_h}{N_h} \right)$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 s_h^2}{n_h} \left(1 - \frac{n_h}{N_h} \right)$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 (N_h - n_h)}{n_h (N_h - 1)} p_h (1 - p_h)$
$\widehat{D(\hat{\theta})}$	$\sum_{h=1}^L \frac{N_h^2 s_{n_h}^2}{n_h} \left(1 - \frac{n_h}{N_h} \right)$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 s_{n_h}^2}{n_h} \left(1 - \frac{n_h}{N_h} \right)$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h (N_h - n_h)}{n_h - 1} p_{n_h} (1 - p_{n_h})$

Методи за одређивање обима узорка по стратумима

пропорционални	оптимални	оптимални <i>Neuman</i> -ов
	$C = c_0 + \sum_{h=1}^L c_h n_h$	$C = c_0 + cn$
$n_h = \frac{n}{N} N_h$	$n_h = \frac{(C-c_0) \frac{N_h s_h}{\sqrt{c_h}}}{\sum_{k=1}^L \sqrt{c_k} N_k s_k}$	$n_h = \frac{N_h s_h}{\sum_{k=1}^L N_k s_k} n$
$D(\hat{t}_{prop}) = \frac{N}{n} \left(1 - \frac{n}{N} \right) \sum_{h=1}^L N_h s_h^2$		$D(\hat{t}_{opt}) = \frac{1}{n} \left(\sum_{h=1}^L N_h s_h \right)^2 - \sum_{h=1}^L N_h s_h^2$

Стратификован случајан узорак са понављањем

θ	t	$\bar{x}$	p
$\hat{\theta}$	$\hat{t}_{str} = \sum_{h=1}^L N_h \bar{x}_{n_h}$	$\bar{x}_{str} = \frac{1}{N} \sum_{h=1}^L N_h \bar{x}_{n_h}$	$\hat{p}_{str} = \frac{1}{N} \sum_{h=1}^L N_h p_{n_h}$
$D(\hat{\theta})$	$\sum_{h=1}^L \frac{N_h(N_h-1)s_h^2}{n_h}$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h(N_h-1)s_h^2}{n_h}$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 p_h(1-p_h)}{n_h}$
$\widehat{D(\hat{\theta})}$	$\sum_{h=1}^L \frac{N_h^2 s_{n_h}^2}{n_h}$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 s_{n_h}^2}{n_h}$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 p_{n_h}(1-p_{n_h})}{n_h-1}$

Количничко оцењивање на основу простог случајног узорка без понављања

$$R = \frac{t}{t_y} = \frac{\bar{x}}{\bar{y}}$$

θ	R	t	$\bar{x}$
$\hat{\theta}$	$\hat{R} = \frac{\bar{x}_n}{\bar{y}_n}$	$\hat{t}_R = \hat{R}t_y$	$\bar{x}_R = \hat{R}\bar{y}$
$D(\hat{\theta}) \approx$	$\frac{1-\frac{n}{N}}{n\bar{y}^2} \frac{1}{N-1} \sum_{i=1}^N (x_i - Ry_i)^2$	$\frac{N^2(1-\frac{n}{N})}{n} \frac{1}{N-1} \sum_{i=1}^N (x_i - Ry_i)^2$	$\frac{1-\frac{n}{N}}{n} \frac{1}{N-1} \sum_{i=1}^N (x_i - Ry_i)^2$
$\widehat{D(\hat{\theta})} \approx$	$\frac{(1-\frac{n}{N})}{n\bar{y}^2} \frac{1}{n-1} \sum_{i \in S} (x_i - \hat{R}y_i)^2$	$\frac{N^2(1-\frac{n}{N})}{n} \frac{1}{n-1} \sum_{i \in S} (x_i - \hat{R}y_i)^2$	$\frac{(1-\frac{n}{N})}{n} \frac{1}{n-1} \sum_{i \in S} (x_i - \hat{R}y_i)^2$

$$\rho = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{(N-1)s_x s_y} \quad \frac{1}{N-1} \sum_{i=1}^N (x_i - Ry_i)^2 = s_x^2 + R^2 s_y^2 - 2R\rho s_x s_y$$

Количничко оцењивање на основу узорковања јединки са неједнаким вероватноћама избора

θ	R	t	$\bar{x}$
$\hat{\theta}$	$\hat{R} = \frac{\hat{t}_{HT}(x)}{\hat{t}_{HT}(y)} = \frac{\sum_{i=1}^N \frac{x_i}{\pi_i}}{\sum_{i=1}^N \frac{y_i}{\pi_i}}$	$\hat{t}_R = \hat{R}t_y$	$\bar{x}_R = \hat{R}\bar{y}$
$D(\hat{\theta}) \approx$	$\frac{1}{t_y^2} \left[\sum_{i=1}^N \frac{1-\pi_i}{\pi_i} (x_i - Ry_i)^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} (x_i - Ry_i)(x_j - Ry_j) \right]$	$t_y^2 D(\hat{R})$	$\bar{y}^2 D(\hat{R})$
$\widehat{D(\hat{\theta})} \approx$	$\frac{1}{t_y^2} \left[\sum_{i=1}^N \frac{1-\pi_i}{\pi_i^2} (x_i - \hat{R}y_i)^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} \frac{(x_i - \hat{R}y_i)(x_j - \hat{R}y_j)}{\pi_{ij}} \right]$	$t_y^2 \widehat{D(\hat{R})}$	$\bar{y}^2 \widehat{D(\hat{R})}$

Количничко оцењивање на основу стратификованог случајног узорка

Комбиноване оцене

θ	R	t	$\bar{x}$
$\hat{\theta}$	$\hat{R}_c = \frac{\bar{x}_{str}}{\bar{y}_{str}}$	$\hat{t}_{Rc} = \hat{R}_c t_y$	$\bar{x}_{Rc} = \hat{R}_c \bar{y}$
$D(\hat{\theta}) \approx$	$\frac{1}{t_y^2} \left[\sum_{h=1}^L \frac{N_h^2}{n_h} \left(1 - \frac{n_h}{N_h}\right) (s_h^2(x) + R^2 s_h^2(y) - 2R\rho_h s_h(x)s_h(y)) \right]$	$t_y^2 D(\hat{R}_c)$	$\bar{y}^2 D(\hat{R}_c)$

Посебне оцене

θ	t	$\bar{x}$
$\hat{\theta}$	$\hat{t}_{Rs} = \sum_{h=1}^L \hat{R}_h t_h(y) = \sum_{h=1}^L \frac{\bar{x}_{n_h}}{\bar{y}_{n_h}} t_h(y)$	$\bar{x}_{Rs} = \frac{1}{N} \hat{t}_{Rs}$
$D(\hat{\theta}) \approx$	$\sum_{h=1}^L \frac{N_h^2}{n_h} (1 - \frac{n_h}{N_h}) (s_h^2(x) + R_h^2 s_h^2(y) - 2R_h \rho_h s_h(x) s_h(y))$	$\frac{1}{N^2} D(\hat{t}_{Rs})$

Регресионо оцењивање на основу простог случајног узорка без понављања

	$b = b_0$	$\hat{b} = \hat{\rho} \frac{s_n(x)}{s_n(y)}$	
θ	$\bar{x}$	$\bar{x}$	t
$\hat{\theta}$	$\bar{x}_{LR} = \bar{x}_n + b_0(\bar{y} - \bar{y}_n)$	$\bar{x}_{LR} = \bar{x}_n + \hat{b}(\bar{y} - \bar{y}_n)$	$\hat{t}_{LR} = N \bar{x}_{LR}$
$D(\hat{\theta})$	$(1 - \frac{n}{N}) \frac{1}{n} (s_x^2 + b_0^2 s_y^2 - 2b_0 \rho s_x s_y)$	$\approx (1 - \frac{n}{N}) \frac{1}{n} s_x^2 (1 - \rho^2)$	$N^2 D(\bar{x}_{LR})$
$\widehat{D(\hat{\theta})}$	$(1 - \frac{n}{N}) \frac{1}{n} (s_n^2(x) + b_0^2 s_n^2(y) - 2b_0 \rho s_n(x) s_n(y))$		$N^2 \widehat{D(\bar{x}_{LR})}$

Регресионо оцењивање на основу стратификованог случајног узорка

Комбиноване оцене

	$b = b_0$	$\hat{b} = \frac{\sum_{h=1}^L \left(\frac{N_h^2 (1 - \frac{n_h}{N_h})}{n_h (n_h - 1)} \sum_{i \in S_h} (x_{hi} - \bar{x}_{n_h})(y_{hi} - \bar{y}_{n_h}) \right)}{\sum_{h=1}^L \left(\frac{N_h^2 (1 - \frac{n_h}{N_h})}{n_h (n_h - 1)} \sum_{i \in S_h} (y_{hi} - \bar{y}_{n_h})^2 \right)}$
θ	$\bar{x}$	$\bar{x}$
$\hat{\theta}$	$\bar{x}_{LRc} = \bar{x}_{str} + b_0(\bar{y} - \bar{y}_{str})$	$\bar{x}_{LRc} = \bar{x}_{str} + \hat{b}(\bar{y} - \bar{y}_{str})$
$D(\hat{\theta})$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 (1 - \frac{n_h}{N_h})}{n_h} (s_h^2(x) + b_0^2 s_h^2(y) - 2b_0 \rho_h s_h(x) s_h(y))$	
$\widehat{D(\hat{\theta})}$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 (1 - \frac{n_h}{N_h})}{n_h} (s_{n_h}^2(x) + b_0^2 s_{n_h}^2(y) - 2b_0 \hat{\rho}_h s_{n_h}(x) s_{n_h}(y))$	

Посебне оцене

	$b_h = b_{h0}$	$\hat{b}_h = \hat{\rho}_h \frac{s_{n_h}(x)}{s_{n_h}(y)}$
θ	$\bar{x}$	$\bar{x}$
$\hat{\theta}$	$\bar{x}_{LRs} = \frac{1}{N} \sum_{h=1}^L N_h (\bar{x}_{n_h} + b_{h0}(\bar{y}_h - \bar{y}_{n_h}))$	$\bar{x}_{LRs} = \frac{1}{N} \sum_{h=1}^L N_h (\bar{x}_{n_h} + \hat{b}_h(\bar{y}_h - \bar{y}_{n_h}))$
$D(\hat{\theta})$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 (1 - \frac{n_h}{N_h})}{n_h} (s_h^2(x) + b_{h0}^2 s_h^2(y) - 2b_{h0} \rho_h s_h(x) s_h(y))$	$\approx \frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 (1 - \frac{n_h}{N_h})}{n_h} s_h^2(x) (1 - \rho_h^2)$
$\widehat{D(\hat{\theta})}$	$\frac{1}{N^2} \sum_{h=1}^L \frac{N_h^2 (1 - \frac{n_h}{N_h})}{n_h} (s_{n_h}^2(x) + b_{h0}^2 s_{n_h}^2(y) - 2b_{h0} \hat{\rho}_h s_{n_h}(x) s_{n_h}(y))$	

Кластер узорак код кога се примарне јединице бирају као прост случајан узорак

$$t_i = \sum_{j=1}^{M_i} x_{ij} \quad t = \sum_{i=1}^N t_i \quad R = \frac{\sum_{i=1}^N t_i}{\sum_{i=1}^N M_i} \quad \hat{R} = \frac{\sum_{i \in S} t_i}{\sum_{i \in S} M_i}$$

θ	t	$\bar{x}$	t	$\bar{x}$
$\hat{\theta}$	$\hat{t} = \frac{N}{n} \sum_{i \in S} t_i$	$\frac{\hat{t}}{\sum_{i=1}^N M_i}$	$\hat{t}_R = \frac{\sum_{i \in S} t_i}{\sum_{i \in S} M_i} \sum_{i=1}^N M_i$	$\bar{x}_R = \frac{\sum_{i \in S} t_i}{\sum_{i \in S} M_i}$
$D(\hat{\theta})$	$N^2(1 - \frac{n}{N}) \frac{1}{n} \frac{1}{N-1} \sum_{i=1}^N (t_i - \frac{t}{N})^2$	$\frac{D(\hat{t})}{(\sum_{i=1}^N M_i)^2}$	$\approx N^2(1 - \frac{n}{N}) \frac{1}{n} \frac{1}{N-1} \sum_{i=1}^N (t_i - R M_i)^2$	$\approx \frac{D(\hat{t}_R)}{(\sum_{i=1}^N M_i)^2}$
$\widehat{D(\hat{\theta})}$	$N^2(1 - \frac{n}{N}) \frac{1}{n} \frac{1}{n-1} \sum_{i \in S} (t_i - \frac{1}{n} \sum_{j \in S} t_j)^2$	$\frac{\widehat{D(\hat{t})}}{(\sum_{i=1}^N M_i)^2}$	$\approx N^2(1 - \frac{n}{N}) \frac{1}{n} \frac{1}{n-1} \sum_{i \in S} (t_i - \hat{R} M_i)^2$	$\approx \frac{\widehat{D(\hat{t}_R)}}{(\sum_{i=1}^N M_i)^2}$

Кластери са по M секундарних јединица

$$\rho_{uk} = \frac{\sum_{i=1}^N \sum_{j=1}^M \sum_{\substack{k=1 \\ k \neq j}}^M (x_{ij} - \bar{x})(x_{ik} - \bar{x})}{(M-1)(NM-1)s^2}$$

$$D(\hat{t}) = N^2(1 - \frac{n}{N}) \frac{1}{n} \frac{NM-1}{N-1} s^2 (1 + (M-1)\rho_{uk})$$

Кластер узорак код кога се примарне јединице бирају са вероватноћама пропорционалним величини

θ	t	$\bar{x}$
$\hat{\theta}$	$\hat{t}_{HH} = \frac{\sum_{i=1}^N M_i}{n} \sum_{i \in S} \frac{t_i}{M_i}$	$\bar{x}_{HH} = \frac{1}{n} \sum_{i \in S} \frac{t_i}{M_i}$
$D(\hat{\theta})$	$\frac{\sum_{i=1}^N M_i}{n} \sum_{i=1}^N M_i (\frac{t_i}{M_i} - \bar{x})^2$	$\frac{1}{\sum_{i=1}^N M_i} \frac{1}{n} \sum_{i=1}^N M_i (\frac{t_i}{M_i} - \bar{x})^2$
$\widehat{D(\hat{\theta})}$	$\frac{1}{n(n-1)} \sum_{i \in S} \left(\frac{t_i}{M_i} \sum_{j=1}^N M_j - \hat{t}_{HH} \right)^2$	$\frac{1}{n(n-1)} \sum_{i \in S} \left(\frac{t_i}{M_i} - \bar{x}_{HH} \right)^2$

θ	t	$\bar{x}$
$\hat{\theta}$	$\hat{t}_{HT} = \sum_{i=1}^{\nu} \frac{t_i}{\pi_i}$	$\bar{x}_{HT} = \frac{1}{\sum_{i=1}^N M_i} \hat{t}_{HT}$
$D(\hat{\theta})$	$\sum_{i=1}^N \frac{1-\pi_i}{\pi_i} t_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} t_i t_j$	$\frac{1}{(\sum_{i=1}^N M_i)^2} D(\hat{t}_{HT})$
$\widehat{D(\hat{\theta})}$	$\sum_{i=1}^{\nu} \frac{1-\pi_i}{\pi_i^2} t_i^2 + \sum_{i=1}^{\nu} \sum_{\substack{j=1 \\ j \neq i}}^{\nu} \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} \frac{t_i t_j}{\pi_{ij}}$	$\frac{1}{(\sum_{i=1}^N M_i)^2} \widehat{D(\hat{t}_{HT})}$

Систематски узорак

$$N = nk$$

$$s_{sis}^2 = \frac{1}{k(n-1)} \sum_{i=1}^k \sum_{j=1}^n (x_{ij} - \bar{x}_i)^2$$

θ	$\bar{x}$	t
$\hat{\theta}$	$\bar{x}_{sis} = \frac{1}{n} \sum_{i \in S} x_i$	$\hat{t}_{sis} = N \bar{x}_{sis}$
$D(\hat{\theta})$	$\frac{1}{k} \sum_{i=1}^k (\bar{x}_i - \bar{x})^2 = \frac{N-1}{N} s^2 - \frac{k(n-1)}{N} s_{sis}^2 = \frac{N-1}{N} \frac{s^2}{n} (1 + (n-1) \rho_{uk})$	$N^2 D(\bar{x}_{sis})$