Title: Algebraic K-theory of generalized triangular matrix rings.
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Assume A is an associative ring with 1. Quillen defined higher algebraic K–groups for the ring A as the homotopy groups of the 'plus-construction' to $BGL(A)$:

$$K_i(A) = \pi_i(BGL^+(A)), i \geq 1.$$

Remind that also one has $K_0(A)$ which is defined in terms of finitely generated projective left modules over A.

Consider the ring A_2 of 2-by-2 upper triangular matrices over A. In 1974 Quilllen in his lectures in Oberwolfach announced natural isomorphims $K_i(A_2) = K_i(A) \oplus K_i(A)$. Then Dennis and Geller [1] generalized this statement as follows. Let A and B are two associative rings with 1. Assume M is a A-left and B-right bimodule. Then one can form a new ring R of matrices $\begin{pmatrix} a & m \\ 0 & b \end{pmatrix}$ where $a \in A$, $b \in B$, $m \in M$. Dennis and Geller proved that $K_i(R) = K_i(A) \oplus K_i(B)$ for $i = 0, 1, 2$. In [2] Berrick and Keating proved that there is a natural isomorphims $K_i(R) = K_i(A) \oplus K_i(B)$ for all $i \geq 0$. Their original proof was based on investigation of homology groups of $BGL(R)$. In [3] Keating published much shorter proof based on the another (but equivalent) definition of the higher algebric K-groups and the calculus of functors on the category of finitely generated projective left R-modules.

These results by obvious induction argument are true for the ring of upper triangular n-by-n matrices.

In our talk we discuss similar statement for much more general situation. We define a tensor-like uppertriangular structure and define its K_i-group for all i. Then we prove that for all i such K_i-group is the direct sum of K_i-groups of diagonal part of the structure.

References

[1] *R. K. Dennis and S. C. Geller*, K_i of upper triangular matrix rings, Proc. Amer. Math. Soc. 56 (1976), 73-78.

[2] *A.J.Berrick, M.E.Keating*, The K-theory of triangular matrix rings, K-theory, Contemporary mathematics, v. 55, part I, 1986, 69–74

[3] *M. E. Keating*, The K-theory of triangular matrix rings. II, Proc. Amer. Math. Soc. 100 (2) 1987, 235–236