Discrete Morse theory, nonpositive curvature, and simplicial collapsibility

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Abstract

We present different techniques of proving the collapsibility of simplicial complexes which satisfy a nonpositive curvature condition. Among these techniques we shall discuss discrete Morse theory, van Kampen diagrams and the definition of an elementary collapse. Curvature can be expressed both in metric and combinatorial terms. One can either refer to 'nonpositively curved' in the sense of Aleksandrov and Gromov, i.e. by comparing small triangles in the space with triangles in the Euclidean plane. Or, else, one can express curvature combinatorially using a condition, called local 6-largeness, that seems a good analogue of metric nonpositive curvature.