

Topology, geometry and global analysis on manifolds and discrete structures

Abstract:

The project is envisaged as one of the principal coordinators and carriers of multidisciplinary research in Serbia in the area of algebraic topology, differential geometry, global analysis, topological and geometric combinatorics and their applications in discrete and computational geometry and other areas.

One of the goals of the project is to gather together different mathematical disciplines, each with its own techniques, around the same scientific project, focusing on the multidisciplinary study of geometric objects that appear in all of them. These include (symplectic, triangulated, Banach) manifolds, simplicial complexes etc., the objects which are at the center of virtually all methods for construction and analysis of geometric models in mathematics and its applications.

The primary global effect of this multidisciplinary project is a creation of 'critical mass' of scientists for applications of complex techniques (equivariant topology, global analysis, symplectic and contact geometry) to problems of various structures on manifolds, problems of their realization in Euclidean spaces, problems of discretization of geometric objects (cellular structures, oriented matroids, partial orderings, discrete vector fields), etc. The secondary effect is the development of computational topology and discrete and computational geometry, as a vital connection of algebraic topology and geometric combinatorics with contemporary information technologies and applied mathematics.

Project description:

Toric topology with the emphasis on applications in geometry and combinatorics of convex polytopes and connections with algebraic and symplectic geometry of toric varieties (moment map and convexity theorems). A particularly strong international cooperation is planned and expected within this theme (Nigel Ray (Manchester), Victor Buchstaber (Moscow)) with the research topics including K-powers and generalizations, cohomology of quasitoric varieties, non-standard toric actions etc. One of new, potentially fruitful ideas is the use of oriented matroids, as a replacement for matrices with integer entries (vector configuration) in the theory of quasitoric varieties.

Development of the theory of embeddings and immersions of manifolds with added regularity conditions. The emphasis will be on totally skew embeddings (after M. Ghomi and S. Tabachnikov) and neighborly embeddings (after G. Kalai, V. Vassiliev, and others).

Configurations spaces and their compactifications (Fulton-MacPherson, Axelrod-Singer, Kontsevich) with applications on problems of combinatorial geometry. This theme is, within the "configuration map/test space"-scheme, naturally connected with applications of equivariant topological methods on mass partition problems.

The theory of topological partial orders and discrete-continuous polytopes, e.g. the parallel between simplicial complexes and Grassmannian continuous complexes (partial orders).

Let (M, J) be a differentiable manifold with an almost complex structure J . We say that submanifold N of M is slant if the angle between $JTpN$ and TpN , i.e. the angle between tangent space of N and its image under J is independent of point p on N . The constant angle is called Wirtinger angle (or Kahler angle if N

is a surface). The objective is construction of new examples as well as classification of slant submanifolds of certain almost complex manifolds.

Functional calculus for commuting family of unbounded operators on a Banach space. It is a homomorphism from, as wide as possible, algebra of functions defined on R^n to the set of operators on Banach space, such that polynomials are mapped to their usual meaning. In particular, it is investigated the possibility of functional calculus on the algebra of α times differentiable functions, which is slightly smaller than Sobolev space, where α stands for cumulative polynomial growth of the solution of the corresponding abstract Cauchy problem. It is investigated the possibility of the extension of such results to so called non quasy holomorphic functions, as well.

The axiomatic approach to Fredholm theory. Given a C^* algebra it is introduced the family of "finite dimensional" projections which satisfies certain axioms. Further, the Fredholm operator is defined as such operator which in the given representation has closed range, and projections to its kernel and the complement of the range are "finite dimensional". Properties of such Fredholm operators, in particular the index theorem, are investigated. Special cases of this theory are classical Fredholm theory, Fredholm theory on von Neumann algebras in the sense of Atiyah and Singer, as well as Fredholm theory on C^* modules. The possibility of generalization of results about continuity of index and characterization of being semi Fredholm in terms of measure of non-compactness is considered.

Further classification of E_0 semigroups. The index completely classifies E_0 semigroups of type 1, whereas for semigroups of type 2 and 3, the question of full classification is still open. It is only known that there are uncountably many non isomorphic such semigroups. The possibility of construction of E_0 semigroups via multiplication on the space CP^∞ which is lifted to the base Hilbert space is considered. It is also considered, whether obtained E_0 semigroups are mutually isomorphic or not, as well as whether obtained construction can lead to new computable invariants.

Research relevance:

One of the project's objectives is to maintain its position of one of the leading Serbian projects in the area of geometry, topology, global analysis, geometric, topological, and algebraic combinatorics, discrete and computational geometry, and their applications.

An important role of the project is coordination of research in important areas of mathematics which require and critically depend on applications of complex, multidisciplinary mathematical techniques. The areas of mathematics covered by the project are typically developed in well established mathematical centers worldwide, where a required level of expertise and a critical mass of researchers can be maintained.

For illustration the word "homology" (one of the project's key words) in mathematics and its applications symbolizes development and implementation of algebraic and geometric methods for analysis of global phenomena. The techniques of homological algebra, which originated in algebraic topology and today serve as an important part of any major mathematical discipline, have found new applications in the form of so called "persistent homology" to the areas like topological data analysis (computer vision, data mining), computer graphics, robotics, structural biology, chemistry, etc.

The project is one of the leading proponents of research in Serbia in applications of homological and topological methods in problems of computational topology and geometry. This includes: (a) analysis of the topological complexity of problems of cooperative or competitive division (partition) of geometric objects, (b) coincidence and fixed point results (say as a general method for testing if a dynamical system admits equilibrium states), (c) construction and analysis of geometric "configuration spaces" as mathematical models for states of a physical system, "state space" of an algorithm, etc.

The research in the project is coordinated by teams which have established and maintained an intensive international collaboration and exchange of ideas and people with some of the leading centers in Europe in

its area of expertise. This includes Technical University in Berlin (Günter Ziegler), University of Manchester (Nigel Ray), Steklov mathematical institute in Moscow (Victor Buchstaber), etc. The individual researchers and research teams are gathered together around some of the seminars with the longest tradition in Serbia in respective areas (for example the CGTA – seminar, or the Seminar for Combinatorics in Geometry, Topology, and Algebra, as a joint seminar of Faculty of Mathematics of Belgrade University and Mathematical Institute of the Serbian Academy, has been running continually for 25 years). The project also maintains a very close contact and cooperation with thematically complementary project lead by Vladimir Dragovic (Mathematical Institute SASA) and as a joint effort these two projects have founded a local center of excellence "Dynamical Systems, Geometry and Combinatorics", <http://www.mi.sanu.ac.rs/dsgc/dsgc.htm>. The center has been a coordinator of both national and international projects, including the domestic project "Ziva matematika" for popularization of mathematics and training of young scientists.

Slant submanifolds are important class of submanifolds that generalizes almost complex and totally real submanifolds (their Wirthinger angles are 0 and 90 degrees respectively).

Many distinct representations of functions of operators or their families have many applications in various fields, such as norm estimating of the solutions of differential equations, estimates of resolvent or ascent of an operator.

Fredholm theory plays a significant role in contemporary mathematics and has applications in various fields, from integral equations to algebraic topology and non-commutative geometry.

E_0 semigroups are investigated from mid 30's of the previous century for the purpose of quantum mechanics, and still have applications out of mathematics.